DOI: https://doi.org/10.66118/MMP.2025.008

ADAPTIVE CONTROL UNDER UNCERTAINTY

Anishchenko A., Khrustalova S., Timofeyev V.

Rapid progress of computing technology and significant increase of microprocessor performance open new opportunities for real-time control of complex objects. First of all, we are talking about adaptive control systems in the broad sense, which use in real time the incoming information about signals and parameters of the object under study to develop control actions under conditions of significant uncertainty about the object and disturbances acting on it.

One of the fundamental problems of control theory is the problem of control, i.e., determining or estimating the system parameters at different points in time. Knowledge of system parameters plays an important role in system theory when solving control, observation, forecasting and diagnostic problems [2].

The task of identification and the closely related task of control and detection of changes in the properties of an object consists in the construction of an optimal in some sense model of the system (object) based on the results of observations of input and output coordinates of the system, i.e., a formalized representation of this system [3, 4,] and the detection of changes (control) using this model.

In practice, the so-called identification problem in a narrow sense often arises. The identification problem in this sense consists in estimating the parameters and state of the system based on the results of observations of input and output variables obtained under conditions of normal functioning of the object. In this case, the structure of the system is known and the class of models to which the object belongs is specified.

A priori information about the object and its operating conditions is often absent or very poor, so additional tasks have to be solved beforehand. These tasks include: studying the nature and character of external influences, estimation of noise boundaries, early technical diagnostics and control of the object. In this case, control implies recognizing the state of the technical system and detecting changes in the properties of the functioning object [5].

The proposed work solves the problem of identification, control and management of dynamic stochastic systems under conditions of normal operation and significant a priori uncertainty about the noise affecting the object (there is a complete lack of information about the nature and character of the noise) on the basis of an unconventional adaptive approach.

1. Mathematical models of dynamic stochastic objects

Dynamic objects are described by differential, integral and functional equations with respect to some coordinates characterizing their state [5]. The use of computational tools to control such objects leads to the necessity to describe the studied objects by differences or differential equations.

Linear dynamic stochastic objects (Fig. 1) in general case can be represented by a linear difference equation:

$$y(k)+\sum_{i=1}^{n} a_i y(k-i)=\sum_{i=0}^{m} b_i u(k-i)+\sum_{i=0}^{l} c_i w(k-i), \tag{1}$$

where $y(k)$ – output signal, $u(k)$ – input signal, $w(k)$ – perturbation (disturbance); $a_i\,(i=1,...,n)$, $b_i\,(i=0,1,...,m)$, $c_i\,(i=1,...,l)$ – object parameters to be determined; $k=0,1,2,...$ – discrete time.

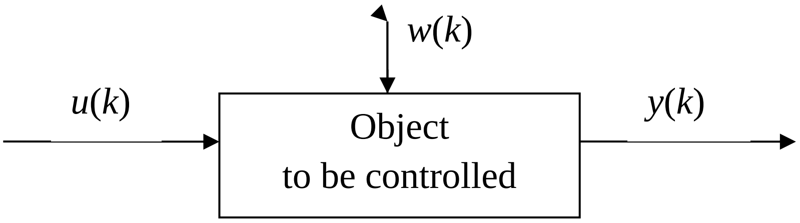

Fig. 1. Linear dynamic stochastic object in general form

Introduction of the delay operator q^{-1}, defined in accordance with the condition

$$q^{-d}y(k)=y(k-d), \tag{2}$$

allows to represent equation (1) in the operator form

$$A\left(q^{-1}\right)y(k)=B\left(q^{-1}\right)u(k)+C\left(q^{-1}\right)w(k) \tag{3}$$

or

$$y(k)=K_u\left(q^{-1}\right)u(k)+K_w\left(q^{-1}\right)w(k), \tag{4}$$

where

$$K_u\left(q^{-1}\right)=B\left(q^{-1}\right)\Big/A\left(q^{-1}\right), \qquad K_w\left(q^{-1}\right)=C\left(q^{-1}\right)\Big/A\left(q^{-1}\right) \tag{5}$$

are the transfer functions of the object under control and perturbation, respectively.

If the dynamic object is stable (and below this assumption is used everywhere), then the roots of the characteristic polynomial are as follows

$$A\left(q^{-1}\right)=1+a_1q^{-1}+a_2q^{-2}+...+a_nq^{-n}$$

lie inside the unit circle centered at the origin.

Moreover, the dynamic object (1) is minimal-phase if the roots of polynomials

$$B\left(q^{-1}\right)=b_0+b_1q^{-1}+b_2q^{-2}+\ldots+b_mq^{-m},$$

$$C\left(q^{-1}\right)=c_0+c_1q^{-1}+c_2q^{-2}+\ldots+c_lq^{-l}$$

lie inside the unit circle centered at the origin, and non-minimal-phase otherwise.

The task of structural identification consists in determining the order of the polynomials used in the description. The parametric identification task consists in determining the parameters of polynomials whose order is already known or estimated. In this case, all parameters to be determined, are included in some vector of unknown parameters.

$$y(k)=K_u\left(q^{-1},\Theta\right)u(k)+K_w\left(q^{-1},\Theta\right)w(k). \tag{6}$$

Equations (1), (3), (4) are the most general and cover as special cases various objects and processes (regression, autoregression, moving average, etc.).

Let us focus on some common ways of describing dynamic objects. Let's consider an autoregressive model to describe dynamic objects. Autoregressive models or ARX models have been investigated in [6]. ARX models are described in the form of the following difference equation

$$y(k)+a_1y(k-1)+\ldots+a_ny(k-n)=b_1u(k)+\ldots+b_mu(k-m)+w(k) \tag{7}$$

or

$$A\left(q^{-1}\right)y(k)=B\left(q^{-1}\right)y(k)+w(k). \tag{8}$$

In equation (5), white noise enters as an unobservable quantity. If we denote «and», we can see that (8) coincides with (6).

To represent a nonstationary dynamic object by an ARX model, the following relation is used:

$$\begin{aligned} &y(k)+a_1(k)y(k-1)+\ldots+a_n(k)y(k-n)= \\ &=b_1(k)u(k-1)+\ldots+b_m(k)u(k-m)+w(k) \end{aligned} \tag{9}$$

or

$$A\left(q^{-1},\Theta(k)\right)y(k)=B\left(q^{-1},\Theta(k)\right)u(k)+w(k). \tag{10}$$

This model contains a time-dependent vector of adjustable parameters of the form

$$\Theta(k)=\left(a_1(k),\ldots,a_n(k),b_1(k),\ldots,b_m(k)\right)^T. \tag{11}$$

By labeling

$$\varphi(k)=\left(-y(k-1),\ldots,-y(k-n),u(k-1),\ldots,u(k-m)\right)^T, \tag{12}$$

we can rewrite (10) as follows:

$$y(k)=\Theta^T(k)\varphi(k)+w(k). \qquad (13)$$

The obtained relation (13) describes a model of pseudo-linear regression type, for the study of which it is possible to apply developed methods of linear regression analysis [6]. It should be noted that the description of a linear dynamic object in the form of (13) is currently widespread, as it allows to represent the identification and control algorithm in a fairly simple and visual form.

Along with autoregressive models, autoregressive moving average models (ARMAX-models) are widely used. ARMAX-models were introduced to solve system identification problems by Ostrem and Bolin [7] and since then such models are among the most popular structures. Various modifications of ARMAX-models are given in [8].

The main difference between the ARMAX model and the ARX model (9) is to give more flexibility in describing the properties of the disturbance. In this model, the model error is represented as a moving average of white noise, which leads to the following relation:

$$\begin{aligned}&y(k)+a_1(k)y(k-1)+\ldots+a_n(k)y(k-n)=\\&=b_1(k)u(k-1)+\ldots+b_m(k)u(k-m)+\\&+w(k)+c_1(k)w(k-1)+\ldots..+c_l(k)w(k-l).\end{aligned} \qquad (14)$$

If we denote

$$C\left(k,q^{-1}\right)=1+c_1(k)q^{-1}+\ldots+c_l(k)q^{-l},$$

then equation (14) can be written in the form of

$$A\left(k,q^{-1}\right)y(k)=B\left(k,q^{-1}\right)u(k)+C\left(k,q^{-1}\right)w(k), \qquad (15)$$

which leads to (6) if we put

$$K_u\left(k,q^{-1},\Theta\right)=B\left(k,q^{-1},\Theta\right)\Big/A\left(k,q^{-1},\Theta\right),$$

$$K_w\left(k,q^{-1},\Theta\right)=C\left(k,q^{-1},\Theta\right)\Big/A\left(k,q^{-1},\Theta\right)$$

Then

$$\Theta(k)=\left(a_1(k),\ldots,a_n(k),b_1(k),\ldots,b_m(k),c_1(k),\ldots,c_l(k)\right)^T.$$

In this case, initialization of the estimation procedure at the moment $k=0$ requires knowledge of the following quantities $y(0),\ldots,y\left(-k^*+1\right)$, $k^*=\max(n,l)$, $\hat{y}(0),\ldots,\hat{y}(-l+1)$, where $\hat{y}(k)$ is the estimate of the model output signal.

If these quantities are missing, they can be taken equal to zero, which introduces inaccuracy into the model. It is also possible to start the recursion at the moment $\max\left(k^{*},u(l)\right)$ and include the unknown initial conditions $y(k), k=1,...,l$ in the vector Θ.

Having introduced the notations

$$\varepsilon(k)=y(k)-\hat{y}(k) \tag{16}$$

$$y(k)=\left(-y(k-1),...,-y(k-n),u(k-1),...,u(k-m),\varepsilon(k-1),...,\varepsilon(k-l)\right)^{T} \tag{17}$$

expression (15) can be written as follows:

$$y(k)=\Theta^{T}(k)\varphi(k)+w(k). \tag{18}$$

The variable used in the relationship $\varepsilon(k)$ is called the prediction error or generalized residual. Equation (18) itself is called a pseudolinear regression $\varphi(k)$ due to the nonlinear dependence of the vector on $\Theta(k)$.

The structures discussed above can actually lead to a large number of models, depending on which of the sets A, B, C are included in the model. For convenience, a generalized model structure for describing a non-stationary object can be used:

$$A\left(k,q^{-1}\right)y(k)=\frac{B\left(k,q^{-1}\right)}{\tilde{B}\left(k,q^{-1}\right)}u(k)+\frac{C\left(k,q^{-1}\right)}{\tilde{C}\left(k,q^{-1}\right)}w(k)$$

where

$$\tilde{B}\left(k,q^{-1}\right)=1+\tilde{b}_{1}(k)q^{-1}+...+\tilde{b}_{l}(k)q^{-l};$$

$$\tilde{C}\left(k,q^{-1}\right)=1+\tilde{c}_{1}(k)q^{-1}+...+\tilde{c}_{l}(k)q^{-l}.$$

The variable $\varepsilon(k)$ used in the relationship is called the prediction error or generalized residual. Equation (18) itself is called a pseudolinear regression due to the nonlinear dependence of the vector $\varphi(k)$ on $\Theta(k)$.

The structures discussed above can actually lead to a large number of models, depending on which of the sets are included in the model. A, B, C.

For convenience, a generalized model structure for describing a non-stationary object can be used:

$$A\left(k,q^{-1}\right)y(k)=\frac{B\left(k,q^{-1}\right)}{\tilde{B}\left(k,q^{-1}\right)}u(k)+\frac{C\left(k,q^{-1}\right)}{\tilde{C}\left(k,q^{-1}\right)}w(k), \tag{19}$$

where

$$\tilde{B}\left(k,q^{-1}\right)=1+\tilde{b}_1(k)q^{-1}+\ldots+\tilde{b}_l(k)q^{-l};$$

$$\tilde{C}\left(k,q^{-1}\right)=1+\tilde{c}_1(k)q^{-1}+\ldots+\tilde{c}_l(k)q^{-l}.$$

However, for most practical purposes this structure is too general. In applications, one or more of the five polynomials are usually assumed to be equal to 1. Using the above $\Theta(k)$ vector of generally non-stationary parameters and the generalized history vector $\varphi(k)$ allows us to represent model (19) as a pseudo-linear regression (18). If the input signals $u(k)$ and outputs $y(k)$ are vectors $(p_u \times 1)$ and $(p_y \times 1)$ respectively, then the simplest generalization of the set of models (6) is

$$\begin{aligned}&y(k)+A_1(k)y(k-1)+\ldots+A_n(k)y(k-n)=\\&=B_1(k)u(k-1)+\ldots+B_m(k)u(k-m)+w(k)\end{aligned}, \tag{20}$$

where $A_i(k), B_i(k)$ are the matrices of dimensions $p_y \times p_y$ and $p_y \times p_u$, respectively. Similarly, matrix polynomials of the variable are introduced: of the models (6) there are

$$A\left(k,q^{-1}\right)=A_1(k)q^{-1}+\ldots+A_n(k)q^{-n};$$

$$B\left(k,q^{-1}\right)=B_1(k)q^{-1}+\ldots+B_m(k)q^{-m}.$$

In this case, we get not a vector, but a $\Theta(k)$ matrix of the required parameters of dimension $\left(\left(np_y+mp_u\right)p_y\right)$:

$$\Theta(k)=\left[A_1(k),A_2(k),\ldots,A_n(k),B_1(k),\ldots,B_m(k)\right]^T \tag{21}$$

and

$$\varphi(k)=\left[-y^T(k-1),\ldots,-y^T(k-n),u^T(k-1),\ldots,u^T(k-m)\right]^T - \tag{22}$$

a column vector of dimension $\left(n\times p_y+m\times p_u\right)\times 1$, which allows us to represent (21) by the matrix equation of linear regression (13).

These formulas can be considered as a set of linear regressions written one under the other with the same input vector.

Thus, as follows from the above, the description of a p_y linear dynamic stochastic object by equation (13) is quite general. On the other hand, the representation of the object under study by model (13) allows the use of well-developed methods of regression analysis and methods for identifying static objects for identifying parameters.

These formulas can be viewed as a set of linear regressions written one under the other with the same input vector. However, the application of regression analysis methods requires certain assumptions regarding the statistical properties of useful signals and interference. In particular, it is assumed that the interference is a random Ψ vector with zero mathematical expectation and a diagonal covariance matrix.

$$M\left\{\Psi(k)\right\}=0, \quad i=1,2,...,k, \quad \operatorname{cov}\left\{\Psi((k)\right\}=\sigma_w^2 I. \tag{23}$$

Here $M\{\bullet\}$ – the symbol is the symbol of mathematical expectation,

σ_w^2 – is the variance of noise,

I – is the identity matrix of dimension $k \times k$.

All limitations associated with classical regression analysis are determined by the traditional criterion of squared errors and, accordingly, the least squares method, which is quite rigidly tied to the normal distribution law of variables and disturbances. If, however, it is only known about the interference that it is limited in amplitude

$$\left|w(k)\right| < \delta \ , \tag{24}$$

then other methods are used to estimate the parameters, which do not use any information about the statistical properties of disturbances except for their belonging to a certain limited interval.

Currently, for estimating parameters in the presence of limited-amplitude interference, the most widely used algorithms are those based on the polytope method (and, as a special case, orthotopes), algorithms containing a dead zone, and algorithms based on constructing ellipsoids. This direction is currently developing rapidly and attracts much research.

2. Recursive identification algorithms

We consider an object described by the pseudolinear regression equation (13):

$$y(k)=\Theta^T(k)\varphi(k)+w(k)$$

with noise satisfying constraint (24)

$$\left|w(k)\right| < \delta .$$

Then equation (13) can be rewritten as

$$y(k)=\Theta^T(k)\varphi(k)+w(k)$$

with a vector of sought parameters satisfying the system of inequalities

$$\left|w(k)\right| < \delta .$$

Then equation (13) can be rewritten as

$$\varepsilon(k) = y(k) - \Theta^T(k)\varphi(k)$$

with a vector $\hat{\Theta}(k)$ sought parameters satisfying the system of inequalities

$$\left(y(k) - \Theta^T(k)\varphi(k)\right)^2 \le \delta^2 \quad \forall k = 1, 2, \ldots$$

Only those that belong to the set can be used as estimates.

With the vector of the sought parameters satisfying the system of inequalities.

Only those that belong to the set can be used as estimates:

$$S(k) = \left\{ \Theta : y\left((k) - \Theta^T \varphi(k) \right)^2 \le \delta^2 \right\}. \tag{25}$$

The undefined parameters belong to the domain bounded by the hyperplanes (25). The domain is a simple connected domain. The space is a convex polytope, which can become quite complex if - is large (Fig. 2).

From a geometric point of view, the set $S(k)$ is a monotonically increasing sequence of intersections of convex polytopes $F(k)$

$$S(k) = \bigcap_{j=1}^{k} F_j = S(k-1) \cap F(k) \tag{26}$$

$$F(k) = \left\{ \Theta : \left| y(k) - \Theta\varphi(k) \right| \le \delta \right\}. \tag{27}$$

Calculating estimates $\hat{\Theta} \in S(k)$ is a complex problem, the solution of which can be significantly simplified by constructing a certain set that restricts $S(k)$ [10]. These restrictions can be specified in the form of orthotopes [11]. The problem of constructing an identification algorithm consists of finding a procedure for recurrent refinement based on the existing values when new observations of input and output signals arrive $S(k)$. The region of uncertain parameters, limited by hyperplanes (25), can be approximated by an orthotope whose edges are parallel to the axes. A geometric illustration of this approach is shown in Fig. 3.

The boundaries $\Theta_{\min}$, $\Theta_{\max}$ of the indefinite interval are associated with the i-th parameter and give an idea of the minimum and maximum values of the criterion

$$J_i(\Theta) = \Theta_i, \tag{28}$$

where is the permissible area of parameters Θ is the area $S(k)$.

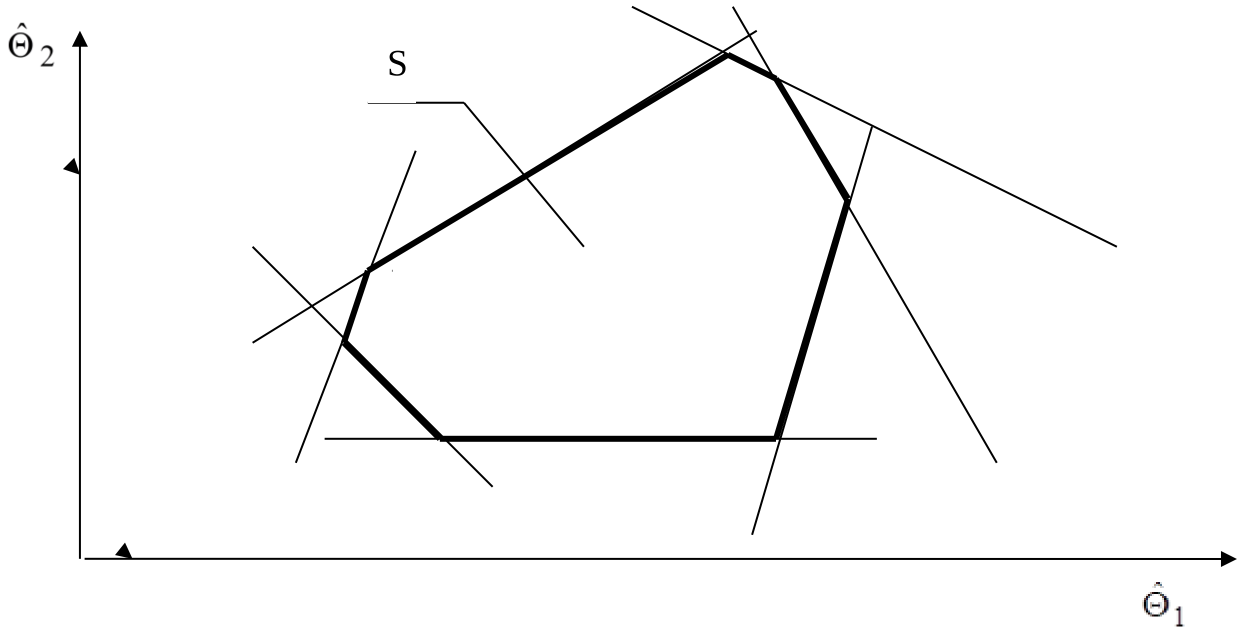

Fig. 2. Finding the range of acceptable values

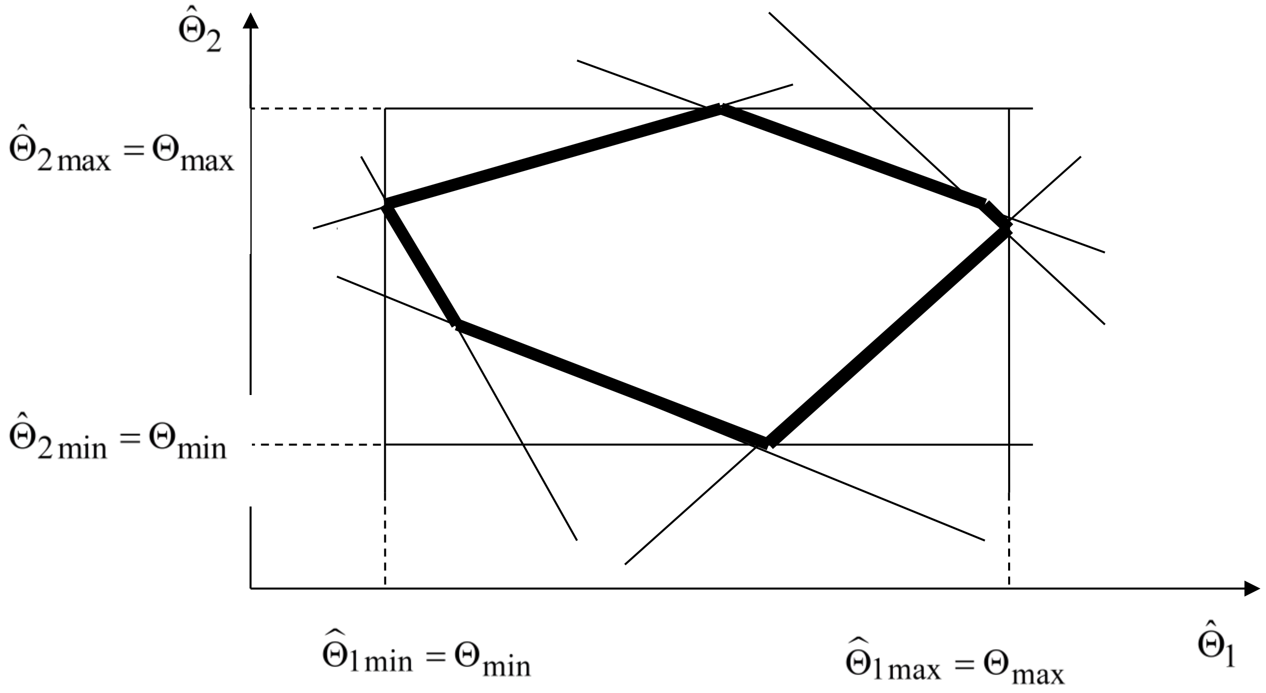

Fig. 3. Approximation of an indefinite interval by an orthotop

Criterion (28) is determined by a set of linear inequalities. Each boundary, therefore, is obtained by solving the problem of linear programming [9, 10]. Calculating the boundaries of the orthotope requires the solution of linear programming tasks, where – the size of the space. Each of the linear programming problems contains linear inequalities. The resulting algorithm is not recurrent and involves a large number of more complex calculations than algorithms of the method of least squares (MHK).

When the area $S(k)$ is narrow and oriented at a large angle to the axes, limiting the axial orthotope can give very inaccurate results. However, it is possible to try to work with an orthotope of minimal volume.).

If it is known about the interference that they satisfy the condition (24), then this information can be taken into account in the algorithm containing the zone of insensitivity. At the same time, the requirements regarding the knowledge of the properties of interference are reduced, but the identification algorithm itself is coarsened.

The idea of using the insensitivity zone in the algorithm is based on the fact that even in the case of accurate determination of the model parameters, there remains an error (mismatch between the output signals of the object and the model), the magnitude of which is determined by the magnitude of the noise (24). On the identification algorithm it will be as follows. The magnitude of the error $\Theta(k-1)=\Theta(k)$ in the case is equal

$$y(k)-\Theta^T(k)\varphi(k)=w(k),$$

as follows. The error value in the case of $\Theta(k-1)=\Theta(k)$ is equal to

$$y(k)-\Theta^T(k)\varphi(k)=w(k),$$

that is, on the $(k-1)$-m step, the correction of the estimate $\Theta(k-1)$ will be made on the value, defined $w(k)$ and on the $(k-1)$-om step, the $\Theta(k-1)$ estimate will be obtained. To prevent this from happening, a zone of insensitivity is introduced, in which case

$$\left|y(k)-\Theta^T(k)\varphi(k)\right|\le\delta(k)$$

that is, on the second step, the correction of the estimate will be made on the value determined and on the second step, the estimate will be obtained. To prevent this from happening, a zone of insensitivity is introduced, in which case

$$\xi(\varphi)=\begin{cases}\varphi-\delta & at \quad \varphi>\delta;\\ 0 & at \quad |\varphi|\le\delta;\\ \varphi+\delta & at \quad \varphi<-\delta,\end{cases}$$

no assessment correction occurs when.

Various algorithms including deadband type nonlinearity were considered in the works [11–13].

If the noise distribution density is strictly limited in the interval $\left(-\delta,\delta\right)$, then these algorithms converge. The estimation procedure itself is reduced to the sequential projection of current estimates $\Theta\left(k-1\right)$ into a system of infinite bands limited by parallel planes and determined by inequalities in the parameter space

$$-\delta \le y\left(k\right)-\Theta^T\left(k-1\right)\varphi\left(k\right)\le\delta .$$

Thus, in [14] the following algorithm was considered:

$$\Theta\left(k\right)=\Theta\left(k-1\right)+\frac{\xi\left(y\left(k\right)-\Theta^T\left(k-1\right)\varphi\left(k\right)\right)}{\left\|\varphi\left(k\right)\right\|^2}\varphi\left(k\right), \tag{29}$$

which can be rewritten as: which can be rewritten a

$$\Theta\left(k\right)=\Theta\left(k-1\right)+\rho\frac{\operatorname{sign}\left|y\left(k\right)-\Theta^T\left(k-1\right)\varphi\left(k\right)\right|}{\left\|\varphi\left(k\right)\right\|^2}\varphi\left(k\right), \tag{30}$$

where is $\rho\left(k\right)=\left|\xi\left(y\left(k\right)-\Theta^T\left(k-1\right)\varphi\left(k\right)\right)\right|$ is the distance from the point $\Theta\left(k-1\right)$ to the strip:

$$-r\le y\left(k\right)-\Theta^T\left(k-1\right)\varphi\left(k\right)\le r$$

where $\rho\left(k\right)=\left|\xi\left(y\left(k\right)-\Theta^T\left(k-1\right)\varphi\left(k\right)\right)\right|$ – distance from the point $\Theta\left(k-1\right)$ to the strip $-r\le y\left(k\right)-\Theta^T\left(k-1\right)\varphi\left(k\right)\le r$.

In [15] the dynamics of the Strip algorithm was studied:

$$\Theta\left(k\right)=\Theta\left(k-1\right)+\gamma w\left(k\right)\frac{y\left(k\right)-\Theta^T\left(k\right)\varphi\left(k\right)}{\left\|\varphi\left(k\right)\right\|^2}\varphi\left(k\right), \tag{31}$$

where

$$w\left(k\right)=\begin{cases}0, & if\quad \left|y\left(k\right)-\Theta^T\left(k-1\right)\varphi\left(k\right)\right|\ge\delta;\\ 1, & if\quad \left|y\left(k\right)-\Theta^T\left(k-1\right)\varphi\left(k\right)\right|<\delta.\end{cases} \tag{32}$$

In the work [16] the problem of constructing an algorithm with a dead zone that minimizes the criterion was considered

$$J\left(\Theta\left(k\right)\right)=\frac{1}{2}\left\|\Theta\left(k\right)-\Theta\left(k-1\right)\right\|^2 \tag{33}$$

under restriction

$$\left|e(k)\right| \le \delta(k) \tag{34}$$

and provided that the a posteriori error $e^0(k)$ satisfies the relation

$$\left(e^0(k) - i^0(k)r(k)i(k)\right) = 0. \tag{35}$$

Here

$$e^0(k) = y(k) - \Theta^T(k)\varphi(k), \tag{36}$$

$$i(k) = \begin{cases} 0, & if \quad \left|e(k)\right| \le \delta; \\ 1, & if \quad \left|e(k)\right| > \delta, \end{cases} \tag{37}$$

$$i^0(k) = sign\left(e(k)\right). \tag{38}$$

As can be seen from (36), the difference between the a posteriori error $e^0(k)$ and $e(k)$ is that when calculating $e^0(k)$ at the k-th step, the estimate $\Theta(k)$ obtained at the k same step is used, and not $\Theta(k-1)$, used in $e(k)$.

In this case, the problem of constructing an algorithm is reduced to minimizing the Lagrange function of the form:

$$J\left(\Theta(k), \lambda(k)\right) = \frac{1}{2}\left\|\Theta(k) - \Theta(k-1)\right\|^2 + \lambda(k)\left(e^0(k) - e^0(k)\delta(k)\right)i(k), \tag{39}$$

where is the Lagrange multiplier $\lambda(k)$.

From the minimum condition it follows

$$\frac{\partial\left(\Theta(k)\lambda(k)\right)}{\partial\Theta} = 0; \qquad \frac{\partial\left(\Theta(k)\lambda(k)\right)}{\partial\lambda} = 0, \tag{40}$$

from which we obtain a system of equations, solving which we find

$$\Theta(k) = \Theta(k-1) + \lambda(k)\varphi(k)i(k), \tag{41}$$

$$\lambda(k) = \begin{cases} \left(e(k) - i(k)r(k)\right) \Big/ \left\|\varphi(k)\right\|^2, & if \quad \varphi(k) \ne 0; \\ 0, & if \quad \varphi(k) = 0. \end{cases} \tag{42}$$

The procedure (41)–(42) is a variation of the Kaczmarz algorithm, implementing a combination of the normalized least squares algorithm and the projection algorithm with a dead zone [18, 19].

The error functions used in the algorithms, taking into account the dead zone, are shown in Fig. 4, 5.

The dead zone shown in Fig. 4 is described by the relations.

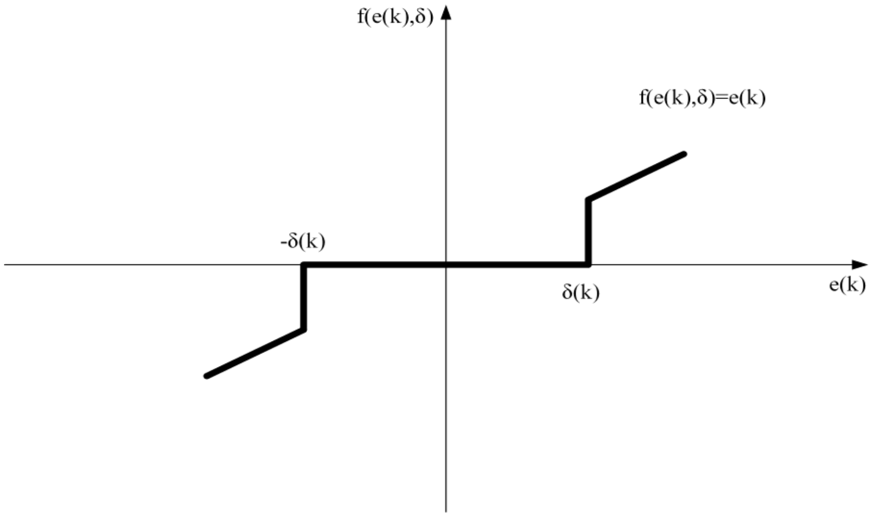

Fig. 4. The dead zone described by (42)

The use of elements implementing the dead zone shown in Fig. 5 and described by the relation has become widespread.

$$f\left(e(k),\delta\right)=\begin{cases} e(k)-\delta(k)\mathrm{signe}(k), & if \quad \left|e(k)\right|\geq\delta; \\ 0, & if \quad \left|e(k)\right|<\delta. \end{cases} \tag{44}$$

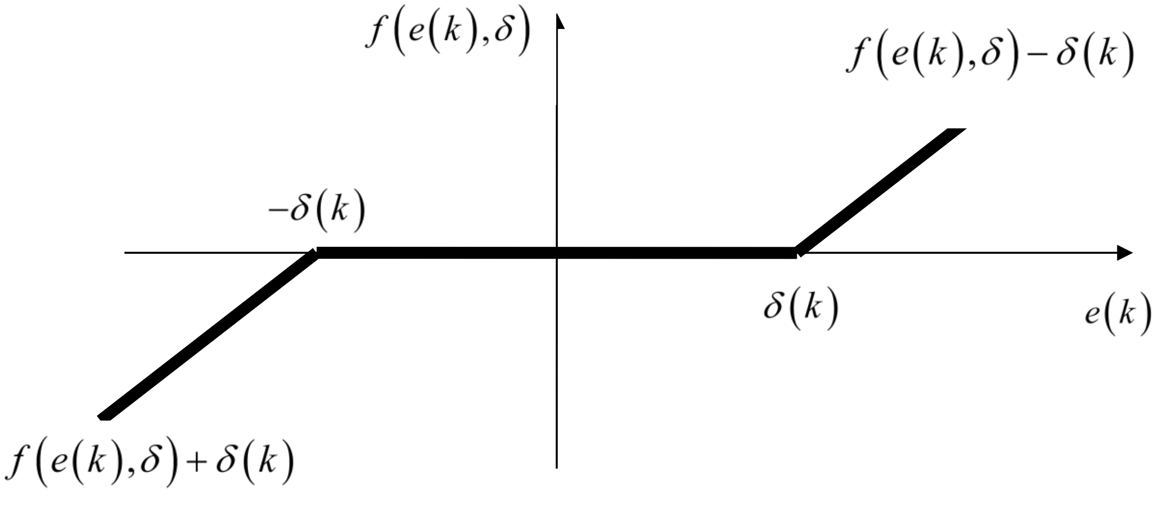

Fig. 5. The dead zone described by (44)

As noted earlier, the calculation of estimates $\Theta\in S(k)$ is a complex problem, the solution of which can be significantly simplified by constructing a certain set, $S(k)$ bounding in the form of ellipsoids [20], the center of which coincides with $\Theta(k)$. In identification problems, algorithms based on the construction of ellipsoids and using the following properties of a unit ellipsoid have become widespread [21]:

$$E=\left\{x\in R^{p}:x^{T}A^{-1}x\leq 1\right\},\ A=A^{T}>0$$

– the axes of the ellipsoid form an orthogonal system of eigenvectors of the A matrix, and the $\lambda_i^{1/2}$ semi-axes are equal to, where are the λ_i eigenvalues $A\left(i=1,2,...,p\right)$;

– the volume of the ellipsoid is proportional to the product of the semi-axes, i.e. proportional to $\sqrt{\det A}$;

– the sum of the squares of the semi-axes $\sum_{i=1}^{k}\left(\lambda_i^{1/2}\right)^2$ is equal to the trace of the matrix A.

Let

$$E\left(k-1\right)=\left\{\Theta:\left(\Theta-\Theta\left(k-1\right)\right)^T P^{-1}\left(k-1\right)\left(\Theta-\Theta\left(k-1\right)\right)\le r^2\right\} \qquad (45)$$

some ellipsoid that contains the intersection, i.e. $S\left(k-1\right)$. Here $S\left(k-1\right)\in E\left(k-1\right)$is the center of the ellipsoid $S\left(k-1\right)\in E\left(k-1\right)$and is the matrix defining the semi-axes $E\left(k-1\right)$ (i.e. the orientation and dimensions of the ellipsoid). The matrix $P\left(k-1\right)$ is symmetrically permutable, and $P\left(k-1\right)=P^T\left(k-1\right)>0$. For degenerate ellipsoids, the constraints correspond to two parallel hyperplanes, $P\left(k-1\right)=P^T\left(k-1\right)>0$ where is determined non-uniquely and is a matrix $P\left(k-1\right)$ with unit rank.

The center of the ellipsoid $\Theta\left(k-1\right)$ is an estimate of the desired parameter vector Θ at time $k-1$. The arrival of new measurements and gives the following value, and the intersection (26) due to the fact that $S\left(k-1\right)\in E\left(k-1\right)$, is completely included in the intersection $E\left(k-1\right)\cap F\left(k\right)$. The new ellipsoid is constructed so that.

$$\left\{E\left(k-1\right)\cap F\left(k\right)\right\}\subset E\left(k\right). \qquad (46)$$

In addition, the new assessment must provide the condition

$$E_k\le E_{k-1}. \qquad (47)$$

Thus, the task of constructing an algorithm consists of finding such a recurrent refinement based $\Theta\left(k\right),P^{-1}\left(k\right),r^2$on the existing values upon $\Theta\left(k\right),P^{-1}\left(k\right),r^2$ receipt of new observations of input and output signals that (46) and (47) are satisfied. A geometric illustration of this approach is shown in Fig. 6.

The ellipsoid $E\left(k\right)$ is constructed with initial conditions $E\left(k\right)$, where I is the identity matrix of size $p\times p$, and is α some sufficiently large number. This means that the initial ellipsoid E_0 is a sphere of radius $\alpha^{1/2}$. The expression for the construction $E(k)$ is as follows [22]:

$$\Theta_0 = \Theta, \qquad P_0 = \alpha I,$$

$$\Theta(k) = \Theta(k-1) + \frac{P(k-1)\left(y(k) - \varphi^T(k)\Theta(k-1)\right)}{1+\varphi^T(k)P(k-1))\varphi(k)}\varphi(k), \tag{48}$$

$$P(k) = P(k-1) - \frac{P(k-1)\varphi(k)\varphi^T(k)P(k-1)}{1+\varphi^T(k)P(k-1)\varphi(k)}, \tag{49}$$

which is identical in structure to the recurrent least squares method (RLSM).

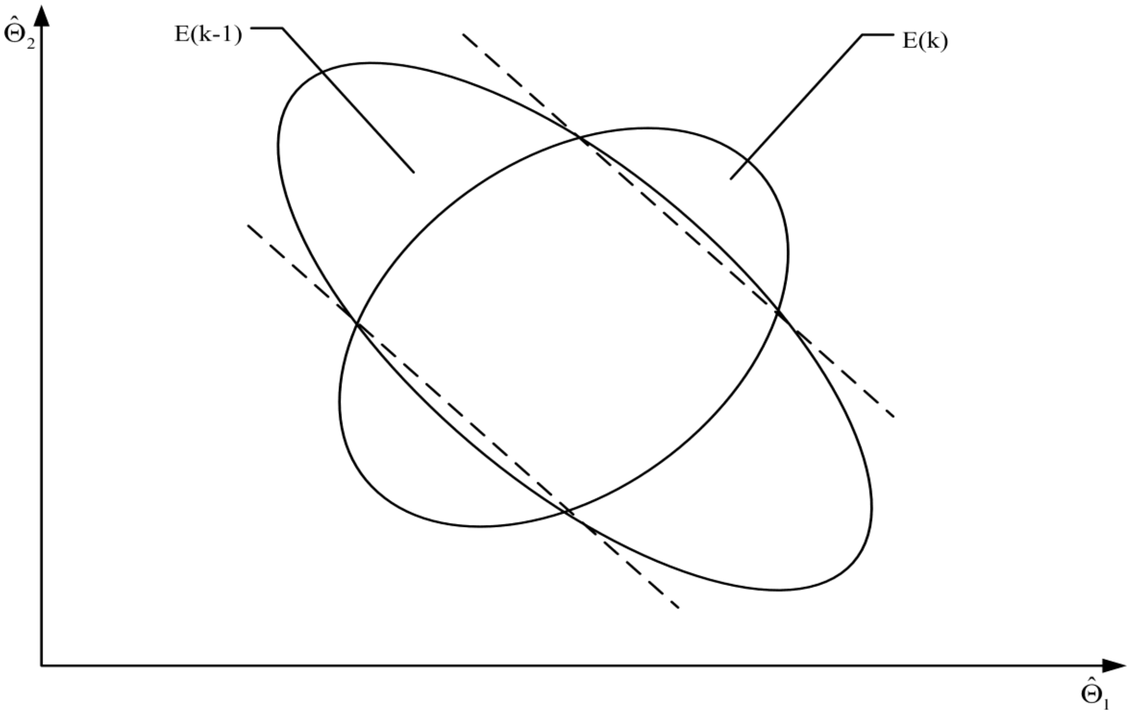

Fig. 6. Ellipsoid constraint

To minimize the geometric dimensions of an ellipsoid $E(k)$, criteria such as minimum volume or minimum trace are used. The resulting algorithms are called, respectively, the «minimum volume algorithm» (or «volume-minimizing algorithm») and the «minimum trace algorithm» (or «trace-minimizing algorithm»).

In [22], a recurrent procedure for calculating an ellipsoid $E(k)$ was propose

In the work [22], a recurrent procedure for calculating the ellipsoid was proposed. To minimize the geometric dimensions of an ellipsoid, criteria such as minimum volume or minimum trace are used. The resulting algorithms are called, respectively, the «minimum volume algorithm» (or «volume-minimizing algorithm») and the «minimum trace algorithm» (or «trace-minimizing algorithm»).

$$E(k)=\Big\{\Theta:\alpha(k)\big(\Theta-\Theta(k-1)\big)^T P^{-1}(k-1)\big(\Theta-\Theta(k-1)\big)+$$

$$+\beta(k)\big(y(k)-\Theta^T(k-1)\varphi(k)\big)^2\le\alpha^2(k)+\beta(k)r^2\Big\}= \qquad (50)$$

$$=\Big\{\Theta:\big(\Theta-\Theta(k-1)\big)^T P^{-1}(k-1)\big(\Theta-\Theta(k-1)\big)\le r^2\Big\},$$

where $\alpha(k)\in(0,1)$ is the parameter for forgetting outdated information; is the parameter for weighing (selecting) newly incoming information$
where $\alpha(k)\in(0,1)$ – obsolescence parameter;

$\beta(k)\in(0,1)$ – weighting (selection) parameter for newly received information.

The task of constructing an algorithm is reduced to finding the center $\Theta(k)$ of an ellipsoid $E(k)$. After transformations, we arrive at a standard optimal algorithm constructed on the basis of bounded ellipsoids:

$$\Theta(k)=\Theta(k-1)+\beta(k)P(k)\varphi(k)e(k); \qquad (51)$$

$$P(k)=\frac{1}{\alpha(k)}\left(P(k-1)-\frac{\beta(k)P(k-1)\varphi(k)\varphi^T(k)P(k-1)}{\alpha(k)+\beta(k)\varphi^T(k)P(k-1)\varphi(k)}\right); \qquad (52)$$

$$e(k)=y(k)-\Theta(k)\varphi(k). \qquad (53)$$

Various modifications of the algorithm (50)–(53) can be obtained by specifying the rules for setting freely selectable parameters of the algorithm $\alpha(k)$ and $\beta(k)$. In [23], it was proposed to select the parameters as follows:

$$\alpha(k)=1/r^2,\quad \varphi(k)=\lambda(k)/r^2\,. \qquad (54)$$

The variable $\lambda(k)$, which is determined by minimizing the geometric dimensions of the ellipsoid $E(k)$, depends on both the step number and the values of the input variables. In principle, it is possible to accept, but this means that the newly received measures do not carry any new information. In this case, the dimensions of the ellipsoid do not change, and the estimates will not be refined.

The variable $\lambda(k)$, which is determined by minimizing the geometric dimensions of the ellipsoid $E(k)$, depends on both the step number and the values of the input variables. In principle, it is possible to accept $\lambda(k)=0$, but this means that the newly received measurements do not carry any new information. In this case, the dimensions of the ellipsoid do not change and there will be no refinement of the estimates.

Ellipsoidal algorithms have become widespread not only in identification problems, but also in control problems when detecting disorders and monitoring parameter changes. At the same time, these algorithms, due to their numerical cumbersomeness, are poorly suited for real-time operation, and therefore their use in real-time control and management problems seems very problematic. In this regard, it seems appropriate to synthesize identification algorithms that are numerically simple, fast-acting, and suited for real-time operation in a closed loop of an adaptive control system.

3. Control systems for dynamic stochastic objects

Any control object (CO) is characterized by the following main groups of variables:

– variables characterizing the state of the object and the signals at its output, and their totality will be designated by the vector y. These variables in the control process must be maintained at a given level or changed according to a given law described by the vector y. The control accuracy may vary depending on the requirements dictated by the technology and the capabilities of the control algorithm. As a rule, the variables included in the vector are measured either directly or calculated using the mathematical model of the CO;

– variables, by changing which the regulator can affect the technological process for the purpose of control. The totality of these variables will be described by the vector of control actions;

– variables, the changes of which are not related to the impact of the control system. These variables reflect the impact of external conditions on the control object, changes in the characteristics of the process itself, etc. They are called disturbing effects and are denoted by the vector. The vector of disturbing effects, in turn, can be divided into two components, the first of which can be measured, and the second cannot. As a rule, the second component describes random effects (interference) to which any real object is subject.

A linear stochastic control object [20] can be described by the following differential equation

$$\begin{aligned} & y(t)+\alpha_1\frac{dy(t)}{dt}+\ldots+\alpha_n\frac{d^n y(t)}{dt^n}= \\ & =\beta_0\frac{u(t)}{dt}+\beta_1\frac{du(t)}{dt}+\ldots+\beta_m\frac{d^m u(t)}{dt^m}+w(t)+\gamma_1\frac{dw(t)}{dt}+\ldots+\gamma_l\frac{d^l y(t)}{dt^l}, \end{aligned} \tag{55}$$

where $y(t)$, $u(t)$ and $w(t)$ are the output, control, and disturbance signals, respectively, at the time t;
are the orders of the object by output, control, and disturbance, respectively;
are the n, m, l parameters of the object.

Qualitatively, the meaning of any problem in control theory is to select such control effects so that the control object behaves in some desired way, most often such that the output signals would be in some sense close to a given law, i.e. it is required to ensure a minimum of some function of the control error, where there is continuous time.

Control laws based on low-order controllers, such as PID controllers, can provide optimal control of objects usually not higher than the second order. Although in practice PID controllers are often used to control higher-order objects, the quality of control in this case can be far from optimal. In addition, tuning a PID controller requires high qualifications and is most often performed using heuristic rules such as the Ziegler–Nichols rule and the Takahashi rule based on it. At the same time, the importance of effective controller tuning is difficult to overestimate, since even small variations in parameters can lead to a significant change in the type of transient processes, which can lead to overexpenditure of energy resources, a decrease in the safety of technological processes and a deterioration in product quality. Thus, there is a need to develop automated procedures for tuning controllers that would reduce the influence of subjective factors, such as the experience of the operator or engineer performing the tuning and improve the quality of control. Moreover, optimal control can be achieved using controllers whose order is equal to the order of the control amplifier.

Control laws based on low-order controllers, such as PID controllers, can provide optimal control of objects usually not higher than the second order. Although in practice PID controllers are often used to control higher-order objects, the quality of control in this case can be far from optimal. In addition, tuning a PID controller requires high qualifications and is most often performed using heuristic rules such as the Ziegler–Nichols rule and the Takahashi rule based on it. At the same time, the importance of effective controller tuning is difficult to overestimate, since even small variations in parameters can lead to a significant change in the type of transient processes, which can lead to overexpenditure of energy resources, a decrease in the safety of technological processes and a deterioration in product quality. Thus, there is a need to develop automated procedures for tuning controllers that would reduce the influence of subjective factors, such as the experience of the operator or engineer performing the tuning and

improve the quality of control. Moreover, optimal control can be achieved using controllers whose order is equal to the order of the control amplifier.

Adaptive controllers can be divided into two classes.

The first class includes self-optimizing controllers, whose task is to achieve the best control quality for a given optimality criterion and the presence of certain information about the object and its signals.

The adaptation process in control systems with this type of controller occurs in three stages:

1. Identification of the object or control system as a whole.
2. Calculation of the controller.
3. Adjustment of the controller or control system as a whole.

The second group of adaptive controllers includes controllers with a reference model.

Their task is to obtain such a response of the closed control loop to a certain input signal that would be as close as possible to the response to the same signal of the reference model.

This adaptation principle assumes the presence of some measurable external signal (for example, a reference action in a tracking system), and adaptation is performed only in those periods when this signal begins to change. In this case, the adaptation process also consists of:

1. Identification of the object or the control system as a whole.
2. Calculation of the regulator.
3. Adjustment of the regulator or the control system as a whole.

The second group of adaptive regulators includes regulators with a reference model.

Their task is to obtain such a response of the closed control loop to a certain input signal that would be as close as possible to the response to the same signal of the reference model.

Conclusions

The analysis of various literary sources devoted to the solution of control problems allowed us to present the existing approaches to solving the specified problems and the place of this study in the following diagram. Here, the class of adaptive critical control systems is especially highlighted.

The problem of adaptive identification, control and critical management of dynamic, non-stationary, stochastic objects under conditions of normal operation and significant a priori and current uncertainty about disturbances acting on the object (there is no information about the nature and character of disturbances) is considered using training models and in the presence of various types of restrictions (on amplitudes, rate of change, energy, etc.) of control and output signals, is non-stationary.

The problem of adaptive identification, control and critical management of dynamic, non-stationary, stochastic objects under conditions of normal operation and significant a priori and current uncertainty about disturbances acting on the object (there is no information about the nature and character of disturbances) using training models and in the presence of various types of restrictions (on amplitudes, rate of change, energy, etc.) of control and output signals is considered, is non-stationary. Developed in works [1–7] and summarized in the monograph [8], a new approach to the synthesis of robust methods of estimation, identification and adaptive control of static and dynamic objects under uncertainty is based on the set-theoretic approach to the interpretation of uncertainty. In this case, the main idea of the approach to the formalization of uncertainties of unknown nature is the use of parametric families of multiple estimates that cover the entire space and can be considered as fuzzy multiple estimates of unknown quantities. This idea is used for simultaneous estimation of both unknown parameters of the object and unknown characteristics of unmeasured disturbances included in the extended vector of estimates. The algorithm for adjusting the estimates of the trained model is built in the class of recurrent finitely convergent procedures for solving systems of inequalities [9].

The synthesis of control algorithms themselves is reduced to the problem of stochastic optimization of the mathematical expectation of a certain function or functional that determines the requirements for the desired value of the output signal of the object. An analysis of various literary sources devoted to solving control problems made it possible to present existing approaches to solving these problems and the place of this study in the following diagram. Here, the class of adaptive critical control systems is especially highlighted. The problem of adaptive identification, control and critical management of dynamic, non-stationary, stochastic objects under conditions of normal operation and significant a priori and current uncertainty about disturbances acting on the object (there is no information about the nature and nature of disturbances) is considered using training models and in the presence of various types of restrictions (on amplitudes, rate of change, energy, etc.) of control and output signals, is considered.

It is assumed that the parameters of the control law are a priori unknown and can change unpredictably over time, and external disturbances are of an unknown nature (stochastic, deterministic, chaotic, etc.) and are limited in amplitude. The control goal is considered to be achieved if all inequalities are satisfied.

Thus, the work is the study and solution of the problem of adaptive critical control of dynamic non-stationary objects operating under conditions of significant a priori and current uncertainty, using training models.

To achieve the stated goal, it is necessary to solve the following problems:

– analysis and selection of structures of models, methods, procedures and spaces of signal description for solving the problem of adaptive critical control;

– development of methods and algorithms of critical control for ARMAX objects taking into account different representations of signals acting on the control object, optimization of parameters;

– synthesis of adaptive identification methods that are not based on optimization of analytical evaluation criteria and intended for operation in the circuit of a critical control system;

– search for optimal conditions that ensure maximum performance of identification methods when working with non-stationary objects;

– development and analysis of methods and algorithms of adaptive critical control based on the proposed critical control procedures and adaptive identification methods;

– simulation modeling and experimental studies of synthesized control laws for dynamic objects of various types;

– implementation of adaptive critical control methods.

References

1. Zakian V. New formulation for the method of inequalities // Proc. IEE. – 1979. – 126. – №6. – P. 579–584.
2. Avramenko V.P., Timofeev V.A. An Adaptive Robust Observation Processing System // Pattern Recognition and Image Analysis. – 1991. – Vol. 1, №1. – P. 27–28.
3. Bodyansky E.V., Vorobyov S.A., Timofeev V.A. Adaptive Recognition of the States of Dynamic Object with Periodic Output Signal // Pattern Recognition and Image Analysis. – 1999. – Vol. 9, №3. – P. 505–509.
4. Bodyansky E.V., Pliss I.P., Timofeev V.A. Discrete Adaptive Identification and Extrapolation of Two-Dimensional Fields // Pattern Recognition and Image Analysis. – 1995. – Vol. 5, №3. – P. 410–416.
5. Bodyansky E.V., Rudenko O.G. Adaptive models in control systems of technical objects. – K.: UMKVO, 1988. – 212 p.
6. Astrom K.J. Lectures on the identification problem- the least squares method Report 6806. Division on Automatic Control, Lund Institute Of Technology / – Lund; Sweden, 1568. – I64 p.

7. Ostrom K., Bolin T. Digital identification of dynamic systems based on normal operating mode data // Proc. of the II International IFAC Symposium on Self-Adjusting Systems. – M.: Nauka, 1969. – P. 99–116.
8. Soderstrom T., Stoic P. System Identification / – New York: Prentice-Hall, 1989. – 612 p.
9. Schweppe F. C. Recursive state estimation: unknown but bounded errors and system inputs // IEEE Trans. on Autom. Contr. – 1968. – 13. – №2. – P. 22–29.
10. Walter E., Piet-Lahanier H. Estimation of parameter bounds from bounded-error data: a survey // Mathematics and Computers in Simulation. – 1990. – P. 449–456.
11. Fomin V.N. Mathematical theory of trained recognition systems. – K.: Publ. KSU, 1976. – 235 p.
12. Derevitsky D.P., Fradkov A.L. Applied theory of discrete adaptive control systems. – K.: Nauka, 1982. – 216 p.
13. Yadykin I.B., Shumsky V.M., Ovsepyan F.A. Adaptive control of continuous technological processes. – K.: Naukova Dumka, 1985. – 240 p.
14. Bunich A.L. Rapidly converging algorithm for identifying a linear object with limited interference // Automation and Telemechanics. – 1983. – No. 8. – P. 101–107.
15. Bedelbaeva A.A. Relay estimation algorithms // Automation and Telemechanics. – 1978. – №1. – P. 87–95.
16. Parkum J.E., Poulsen N.K., Hoist J. Recursive forgetting algorithms / Int. J. Control. – 1992. – 55. – №1. – P. 109–128.
17. Kimura Y., Furukawa T., Kubota H. Generalization of the orthogonal projection algorithm into multidimensional space // Int. J. Modelling and Simulation. – 1993. – 13. – №1. – P. 35–38.
18. F.M. Al-Sadi, Petrova R.V, Timofeev V.A. Algorithms for estimating regression models in the presence of limited noise // Problems of bionics. – 2000. – Issue. 52. – P. 103–107.
19. Agadzhanov S.G., Rudenko O.G., Terenkovskii I.D. Modification of least squares algorithms for identifying parameters in the presence of limited noise // ACS and automation devices. Collection of scientific papers. – Kharkov: KhTURE. – 1997. – Issue. 105. – P. 16–21.
20. Schweppe F. C. Recursive state estimation: unknown but bounded errors and system inputs // IEEE Trans. on Autom. Contr. – 1968. – 13. – №2. – P. 22–29.
21. A1-Sunni F., Al-Nemer T. Fuzzy logic based PID self-tuning scheme for process control // Proc. IFAC-IFIP-IMACS Conf. – Belfort, France, 1997. – P. 185–195.
22. Schweppe F. S. Uncertain Dynamical Systems. – Englewood Cliffs, N.J.: Prentice Hall. – 1973. – 533 p.
23. Fogel E., Huang Y.F. On the value of information in system identification – bounded noise case // Automatica. – 1982. – 18. – No. 2. – P. 229–238.